Stokes' law in complex liquids

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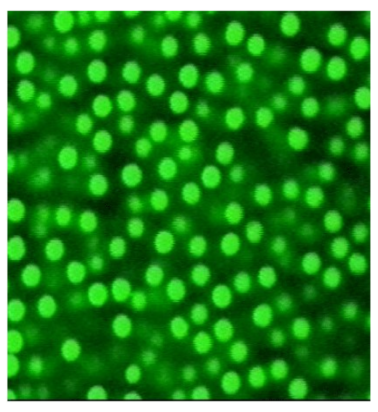
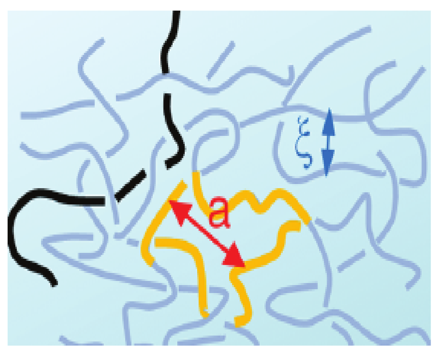
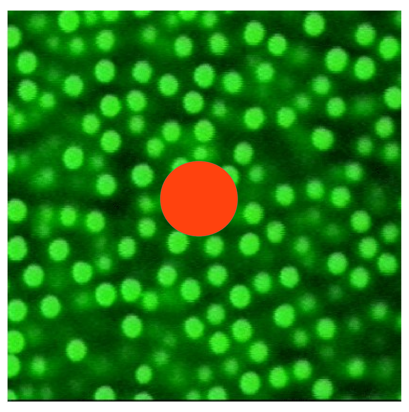
Motivation and goals

Example of complex liquids: suspensions, polymer solutions







Drag force on a spherical particle moving in a complex liquid:

$$\mathbf{F} = \zeta\left(a\right)\mathbf{U}$$

Friction coefficient

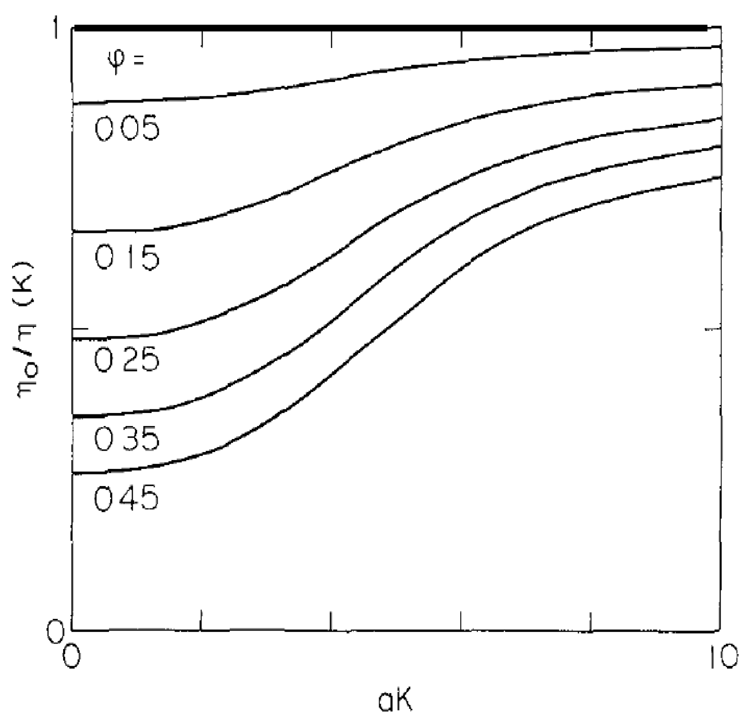
Stokes law in simple liquids:

$$\zeta\left(a\right) = 6\pi\eta_0 a$$

What is the friction experienced by a particle moving in complex liquid?

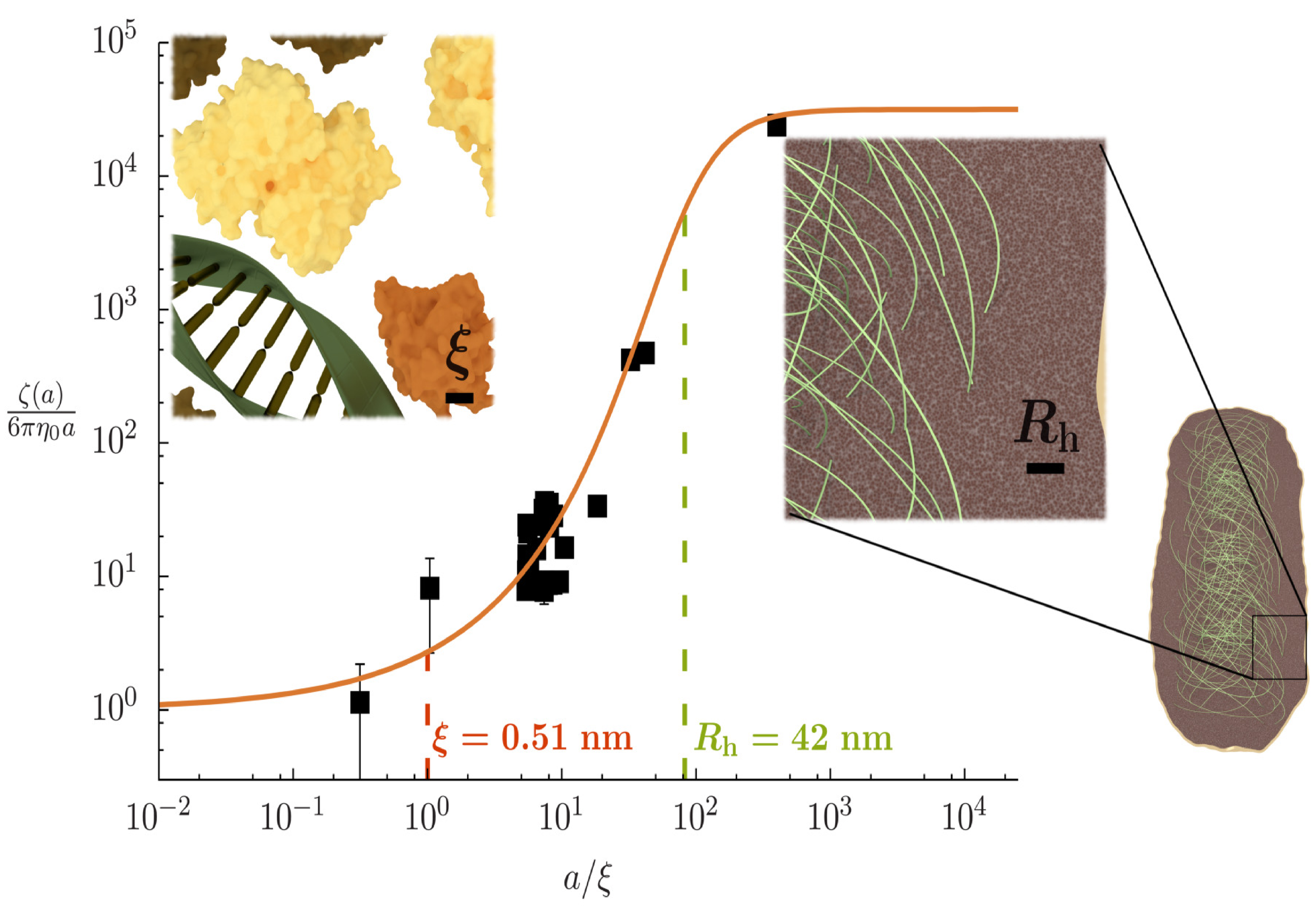
Simple and complex liquids

simple liquid:	complex liquid:
$\eta = \mathrm{const}$ →	$\eta(k)$
Second order partial diff. Eqs.	Infinite order partial diff. Eqs.
$\eta\triangle$ →	$\eta\left(\triangle\right)\triangle$



Beenakker C., Physica A, 128, 48 (1984)

Motion in complex liquids



Experimental data from the literature on diffusion inside Escherichia Coli cell cytoplasm. Normalized friction coefficient as a function of radius of the probe particle [Kalwarczyk et al., *Nano Lett.* 2011, 11, 2157–2163].

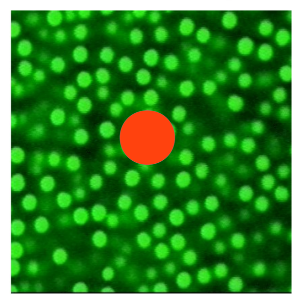
Size of a probe particle is crucial in order to determine the friction experienced by the particle.

Theory

Average velocity field around the probe particle in complex liquid

[Szymczak, P. Cichocki, B. Journal of Statistical Mechanics: Theory and Experiment, 2008, P01025]

$$\langle \mathbf{v}(\mathbf{r})\rangle = \int d^3r'\ \mathbf{G}_{\mathrm{eff}}(\mathbf{r}-\mathbf{r})\mathbf{t}^{\mathrm{irr}}(\mathbf{r}')\mathbf{F}$$



Effective Green function, does not depend on the probe particle:

$$\hat{\mathbf{G}}_{\mathrm{eff}}(\mathbf{k}) = \frac{1}{\eta\left(k\right)k^2}\left(\mathbf{1}-\hat{\mathbf{k}}\hat{\mathbf{k}}\right)$$

Gives forces in complex liquid induced by the probe particle. Also contains all 'interactions' between complex liquid and the probe particle.

For a simple liquid: $\mathbf{t}^{\mathrm{irr}}(\mathbf{r}) = \frac{1}{4\pi a^2}\delta(|\mathbf{r}|-a)\mathbf{1}$

Boundary conditions on the surface of the probe particle: $\mathbf{v}\left(a\hat{\mathbf{r}}\right) = \mathbf{U}$

Idea of our approach

- As experiments show, size of a probe particle is crucial for determination of the friction which it experiences in complex liquid
- It suggests to neglect all 'interactions' between the probe particle and the complex liquid in the above velocity field:

$$\mathbf{t}^{\mathrm{irr}}(\mathbf{r}) \approx \frac{1}{4\pi a^2}\delta(|\mathbf{r}|-a)\mathbf{1}$$

Results

Stokes' law in complex liquids:

$$\zeta\left(a\right) = 3\pi^2\left[\int_0^\infty dk\ \frac{j_0^2\left(ka\right)}{\eta\left(k\right)}\right]^{-1}$$

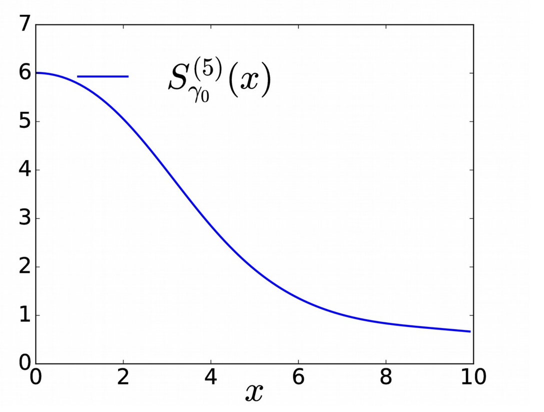
$$j_0\left(x\right) = \frac{\sin\left(x\right)}{x}$$

and its inverse:

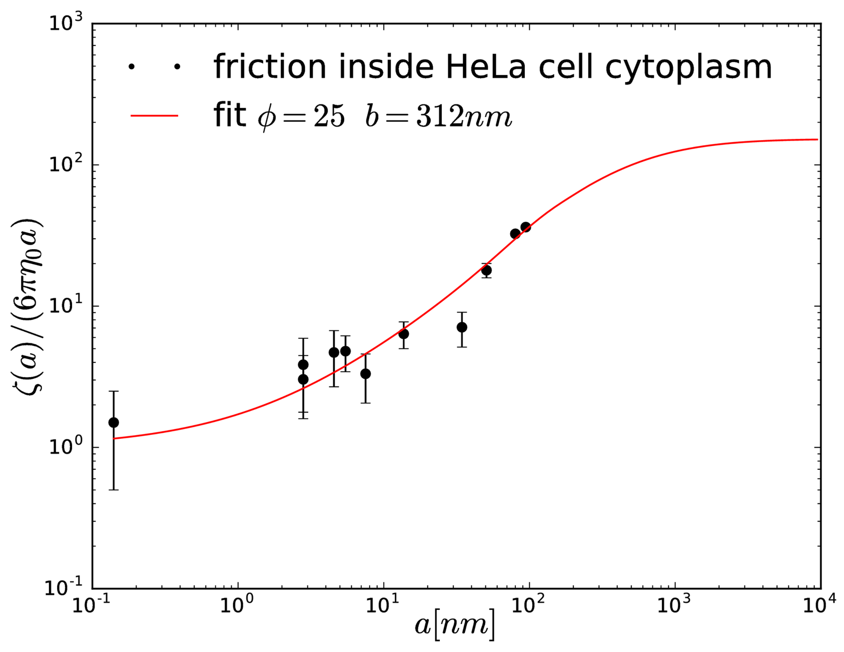
$$\eta\left(k\right) = \frac{1}{48\pi k^2}\left[\int_0^\infty da\ j_0\left(2ak\right)\left(\frac{a^3}{2}\frac{d}{da}\frac{1}{\zeta\left(a\right)} + \frac{a^2}{\zeta\left(a\right)}\right)\right]^{-1}$$

Fitting function for the wave-vector dependent viscosity:
[Beenakker C., Physica A, 128, 48 (1984)]

$$\eta\left(k\right) = \eta_0\left(1+\phi S_{\gamma_0}^{(5)}(kb)\right)$$



First experimental determination of the wave-vector dependent viscosity. We apply the Stokes' law for experimental data for HeLa cell cytoplasm. The viscosity is given by the above fit with $\phi = 25$ $b = 312nm$



Outlook

- Universal relations between translational and rotational friction coefficients
- Generalization to such probe particles as proteins (drug design)
- Active matter

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