# Do bubbles screen?

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# Introduction - bubbles

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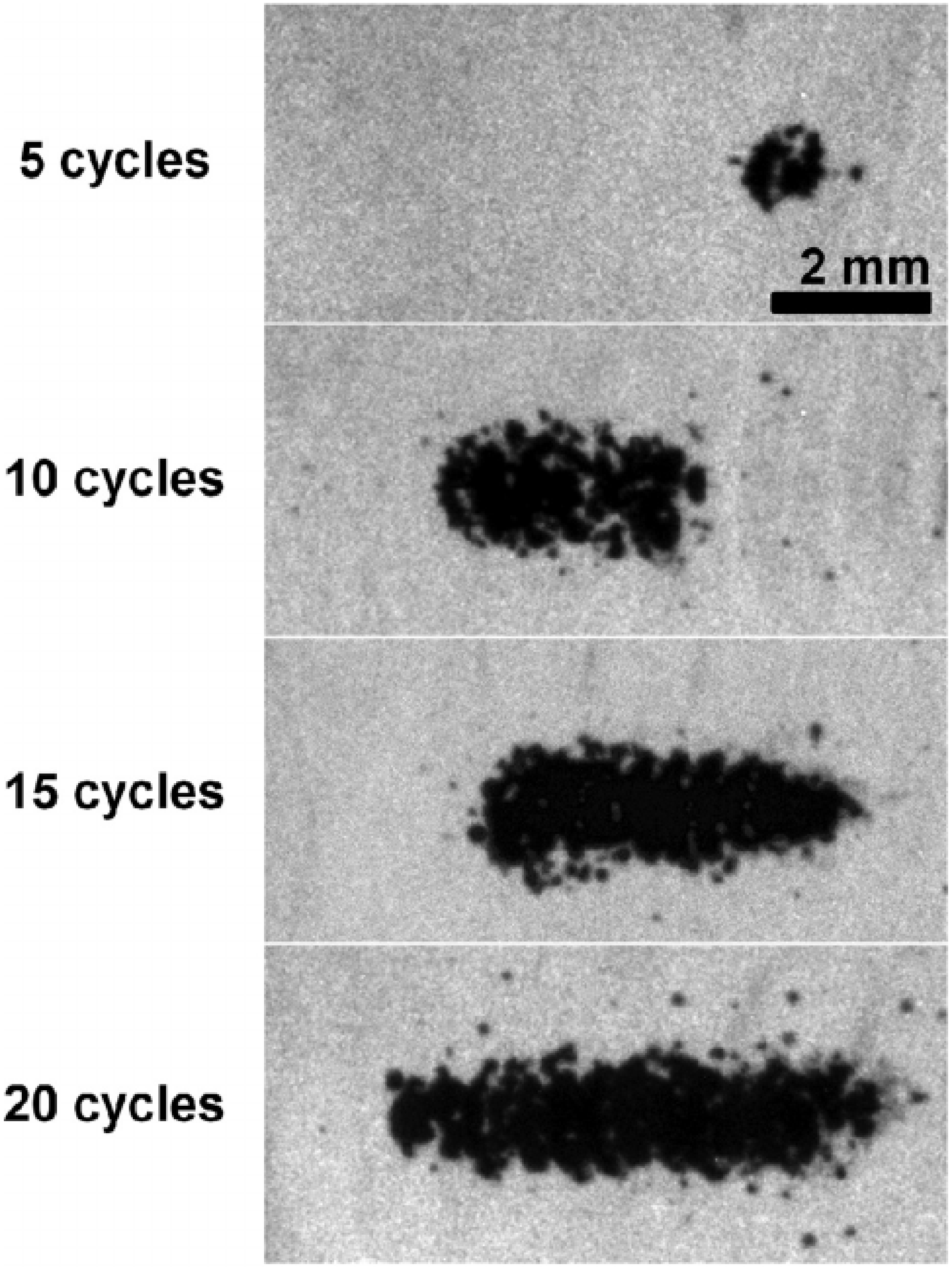



Ultrasound produce growing cloud of bubbles

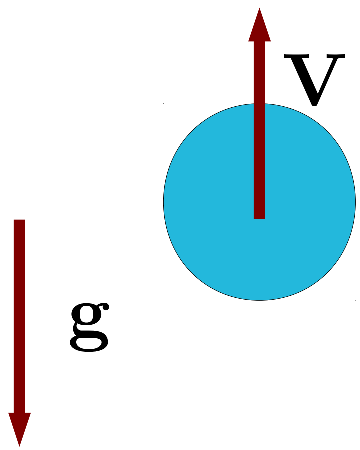
The cloud may mechanically destroy surrounding objects

damage of soft tissue (histotripsy)

Mechanics?

*A. Maxwell et al. J. Acoust. Soc. Am., Vol. 130, No. 4, October 2011*

# Simplified description of liquids with bubbles




Moore, D. *JFM, 1959, 6, 113-13*:
For high Reynolds number

$$Re = \frac{aV}{\eta}$$

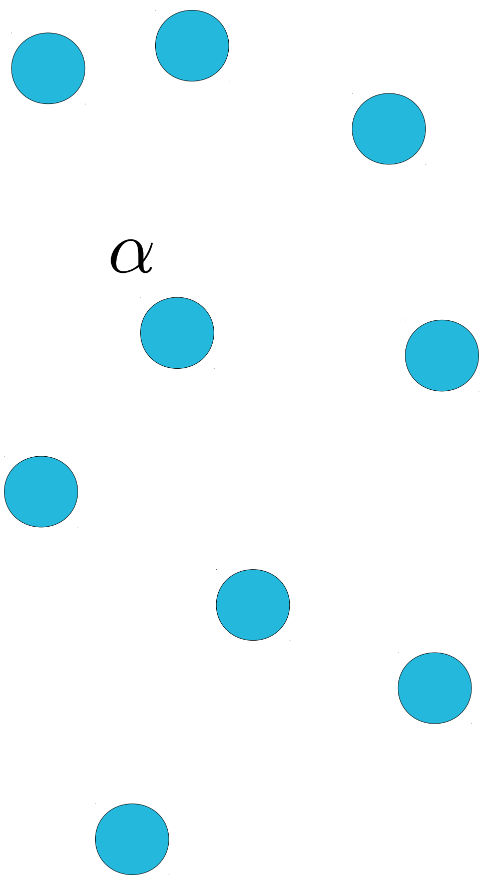
viscosity plays a role only in boundary layer around bubble.

Flow is irrotational

# Simplified description of liquids with bubbles

Configuration of bubbles (radii and positions):

$$X \equiv (a_1, \mathbf{R}_1, \ldots .a_N, \mathbf{R}_N)$$




$$\alpha = 1, \ldots, N$$

$$\nabla \times \mathbf{v} = 0$$

Irrotational flow

$$\nabla \cdot \mathbf{v} = 0$$

incompressible flow

$$\mathbf{v} = \nabla \phi$$

Laplace equation with b.c. on the surfaces of bubbles:

$$\Delta \phi = 0$$

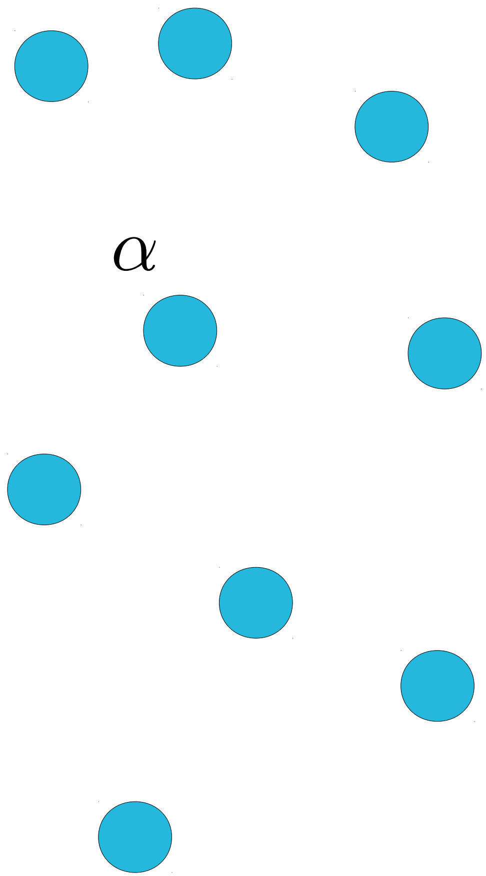
$$\hat{n}(\mathbf{r}) \cdot \nabla \phi|_{\mathbf{r} \in \partial \Omega^\alpha} = \dot{a}_\alpha + \hat{n}(\mathbf{r}) \cdot \dot{\mathbf{R}}^\alpha$$

consequences...

Also used to describe bubbles in ultrasound field.
d'Agostino, L. & Brennen, C. E. JFM, 1989, 199, 155-176

# Kinetic energy and dynamics of bubbly liquids

$$X \equiv (a_1, \mathbf{R}_1, \ldots . a_N, \mathbf{R}_N)$$




$\dot{X}$ Motion of surface of bubbles determine motion of the fluid.

Therefore kinetic energy of the fluid determined by:

$$T = \frac{1}{2} \dot{X}^{\alpha} M^{\alpha\beta} \left(\underline{X}\right) \dot{X}^{\beta}$$

Virtual (added) mass matrix

Lagrangian

Hamiltionian

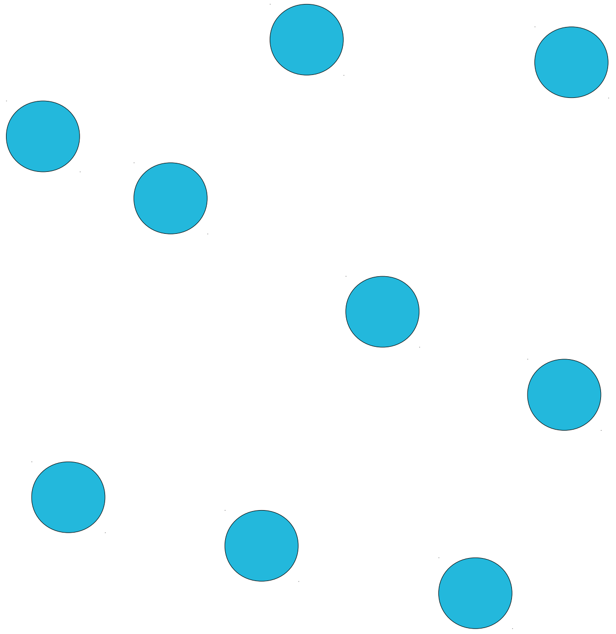
$$H(X, P) = \frac{1}{2} P M^{-1} \left(X\right) P$$

# Virtual mass matrix

$$T = \frac{1}{2}\dot{X}^{\alpha} M^{\alpha\beta}\left(X\right)\dot{X}^{\beta}$$

- Determines kinetic energy of the bubbly liquid
- Determines dynamics of bubbles
- Depends on configuration of all bubbles in liquid
- Determines mechanics - “interactions” of bubbles through liquid

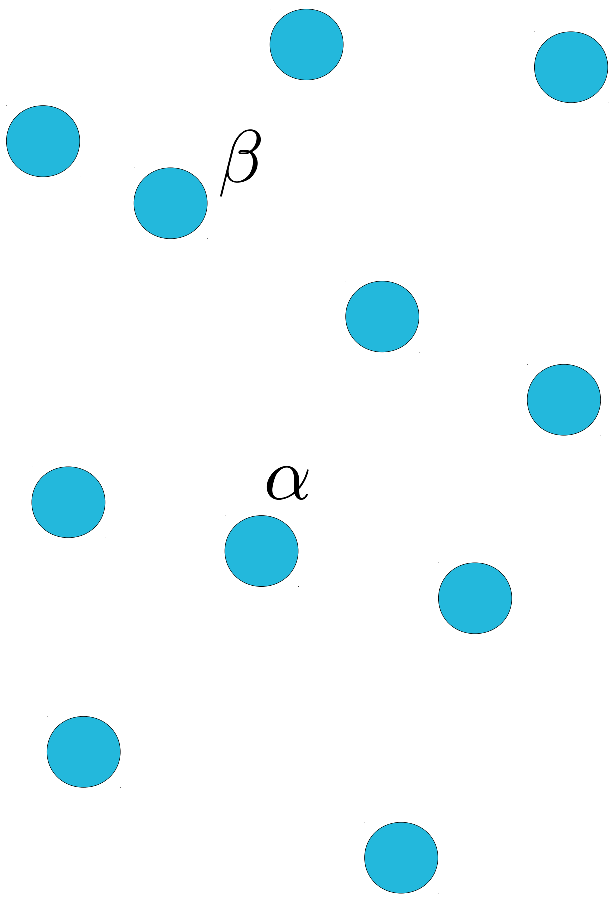
What is the range of interactions described by the virtual mass matrix?



# “Simple” problem

Quiescent fluid, no motion

$$\dot{X} = 0$$




… and pressure (or force) $\mathcal{F}^{\beta}$ appears suddenly on the bubble $\beta$ .

What is acceleration of bubbles $\ddot{X}^{\alpha}$?

The answer follow from equation of motion:

$$\ddot{X}^{\alpha} = [M^{-1}(X)]^{\alpha\beta}\mathcal{F}^{\beta}$$

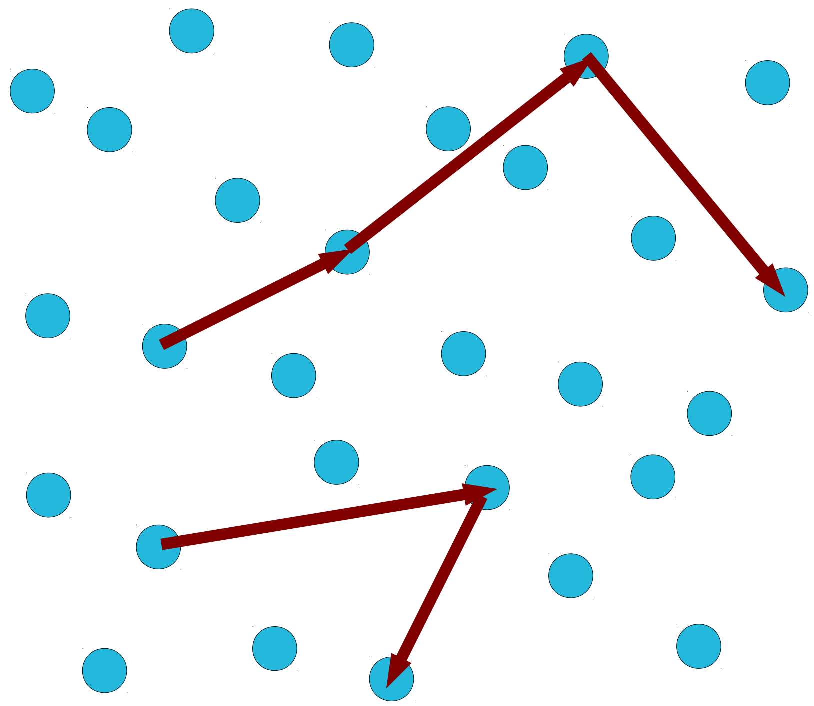
Question about range of interactions, more precisely:
How acceleration of bubble α depends on the distance from bubble β?

# Virtual mass matrix – how to calculate?

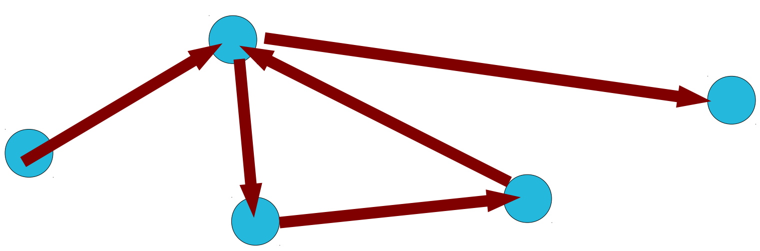
$$\Delta \phi = 0 \quad \longrightarrow \quad M^{\alpha\beta}\left(X\right)$$

*b.c.*

Laplace equation can be solved using method of successive approximations. Interpretation:



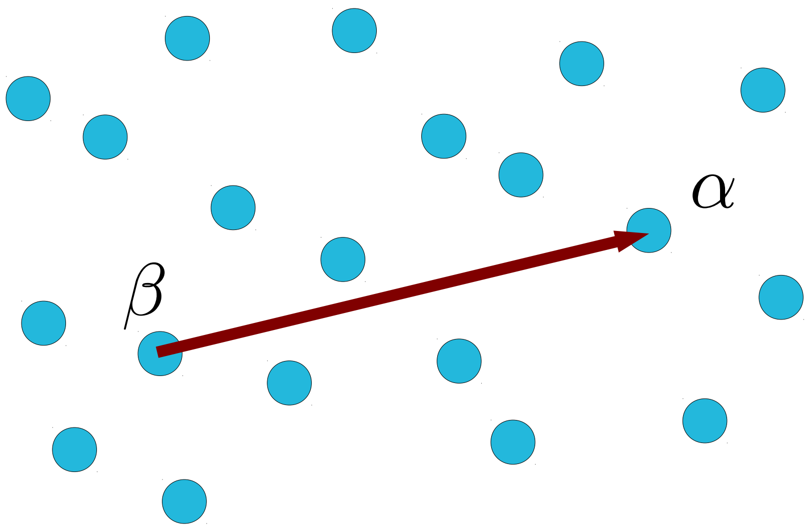
To find virtual mass matrix → sum of all possible paths:



*many-body "interactions"*

# Result

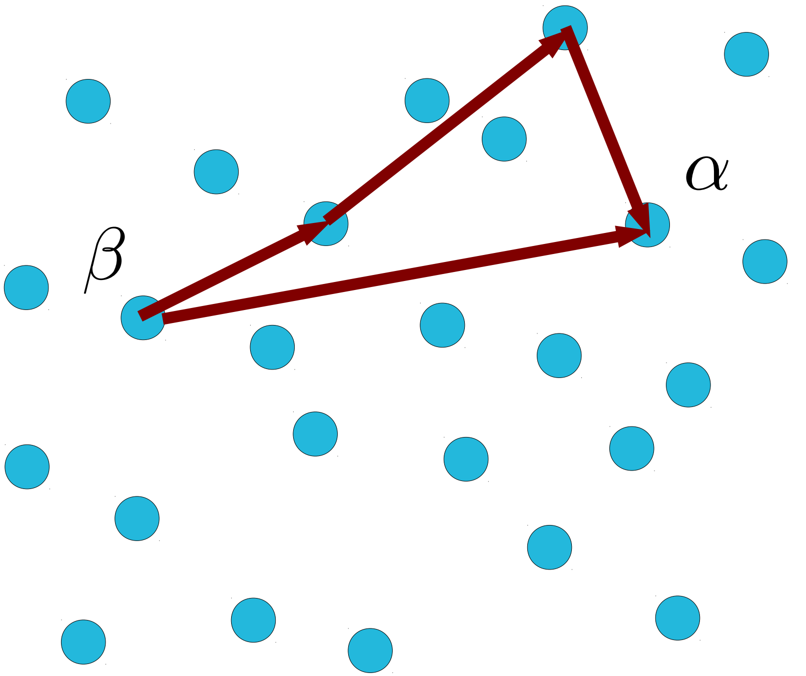
$$\ddot{X}^\alpha = [M^{-1}(X)]^{\alpha\beta}\mathcal{F}^\beta$$




How acceleration depends on the distance from disturbed bubble?

In two body approximation:

$$[M^{-1}(X)]^{\alpha\beta} \sim \frac{1}{|\mathbf{R}^\alpha - \mathbf{R}^\beta|}$$




But with many-body interactions:

$$[M^{-1}(X)]^{\alpha\beta} \sim \frac{e^{-\kappa|\mathbf{R}^\alpha - \mathbf{R}^\beta|}}{|\mathbf{R}^\alpha - \mathbf{R}^\beta|}$$

Screening length:
(electrostatics)

$$\kappa^{-1} = \frac{1}{\sqrt{4\pi a n}}$$

Mechanical screening in bubbly liquids!

# Summary

Bubbles influence their mutual motion, which is described by the virtual mass matrix.

Mechanical screening:

*Two body approximation:*

$$[M^{-1}(X)]^{\alpha\beta} \sim \frac{1}{|\mathbf{R}^\alpha - \mathbf{R}^\beta|}$$

*Rigorous result for cloud of bubbles:*

$$[M^{-1}(X)]^{\alpha\beta} \sim \frac{e^{-\kappa|\mathbf{R}^\alpha - \mathbf{R}^\beta|}}{|\mathbf{R}^\alpha - \mathbf{R}^\beta|}$$

Consequences for:
– effective equations describing bubbly liquids
– histotripsy, ....